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Abstract

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Disciplines

Astrophysics and Astronomy | Physics

Comments

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Multiphoton interactions in lepton photoproduction on nuclei at high energies

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We study multiphoton effects in quantum electrodynamics in lepton photoproduction on heavy nuclei and nuclear medium at high energies. We are interested in energy, charge density, and nuclear geometry dependence of the cross sections. We use the impact parameter representation that allows us to reduce the problem of photoproduction to the problem of propagation of electric dipoles in the nuclear Coulomb field. In the framework of the Glauber model we resum an infinite series of multiphoton amplitudes corresponding to multiple rescattering of the electric dipole on the nucleus. We find that unitarity effects arising due to multiphoton interactions are small and energy independent for scattering on a single nucleus, whereas in the case of macroscopic nuclear medium they saturate the geometric limit of the total cross section. We discuss an analogy between nuclear medium and intense laser beams.

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I. INTRODUCTION

In this article we offer a new perspective on the multiphoton interactions in lepton photoproduction on a heavy nucleus at high energies. Multiphoton effects arise due to multiple rescattering of the projectile photon on the protons of the nucleus. Each scattering contributes a factor of αZ to the cross section and must be taken into account for heavy nuclei. Traditionally, this is accomplished by solving the Dirac equation in the external Coulomb field of the nucleus. An explicit assumption of this approach is that the nucleus can be treated as a pointlike particle. The goal of this article is to solve the photoproduction problem in a general case, by explicitly taking the nuclear size into account. This allows us to evaluate the finite nuclear size effects in lepton photoproduction. However, our main interest is to use the developed formalism to investigate the photoproduction in another extreme case—inside the nuclear medium (e.g. a thin film). We will argue that the nonlinear effects are very different in these two opposite cases. The formalism that we develop and use in this work can be also applied to the very interesting problem of photoproduction on intense laser beams [1]. There the beam size can be tuned so that both limits can perhaps be realized experimentally.

Our approach to the multiphoton effects is based on the Glauber model [2], which allows a proper treatment of the nuclear geometry. In Sec. V and the Appendix we show that the Glauber model calculation of the pair production cross section agrees with the traditional approach pioneered by Bethe and Maximon [3], which is based on a solution of the Dirac equation in an external Coulomb field (see [4–10] for recent developments). This is an important result of this paper. It opens the possibility of a unified approach to the geometric effects in QED. In particular, in this paper we study the dependence of the lepton pair photoproduction on the density of charge sources and nuclear geometry.

The paper is structured as follows. In Sec. II we write the cross section in a mixed coordinate/momentum representation by transforming the scattering amplitude to the transverse coordinate space [11,12]. The cross section then becomes a convolution of the photon wave function, describing splitting of the photon into a lepton-antilepton pair characterized by the transverse vector \mathbf{r} , and the elastic amplitude $i\Gamma^{l\bar{l}Z}$ for the scattering of the lepton electric dipole \vec{l} off the nucleus Z at some impact parameter \mathbf{B} ; see (1). The advantage of this representation becomes clear if we recall that the trajectory of an ultrarelativistic particle in an external field is a straight line. This implies that the dipole representation diagonalizes the scattering matrix. It was shown in [10] that the lepton photoproduction amplitudes take a rather simple form in the dipole representation. This representation was used in [13] for numerical study of the impact parameter dependence of the pair production.

In Sec. III we use the Glauber theory for multiple scattering [2] to express the dipole-nucleus scattering amplitude $i\Gamma^{l\bar{l}Z}$ through the dipole-proton one $i\Gamma^{lp}$. This can be done if the nuclear protons can be considered as independent scattering centers. For large Z their distribution thus follows the Poisson law that leads to a particularly simple expression for the amplitude (10). To avoid possible confusion we emphasize that we consider ultrarelativistic scattering on a “bare” nucleus, i.e. a nucleus stripped of all electrons. Such nuclei are provided by the high energy heavy ion colliders such as RHIC and LHC. Therefore, the problem of screening of the nuclear Coulomb field by electrons—which is an important issue at high energy photon-atom interactions—is not relevant here.

We calculate the scattering amplitude for dipole-proton scattering in Sec. IV. The momentum exchanged in the high energy collision is very small. The corresponding impact parameter is large. The maximal impact parameter b'_{\max} increases with energy. For a single nucleus, it always holds that $b'_{\max} \gg R$. The region of large impact param-

ters is where the dipole cross section picks up its logarithmically enhanced energy dependence. Therefore, b'_{\max} is the effective radius of electromagnetic (EM) interaction of the photon with the nucleus. This is very different from the strong interactions where the Yukawa potential drops exponentially at distances $\sim 1/(2m_\pi)$ much shorter than the nucleus radius.

To make connection with the world of high photon densities, we consider photoproduction off a nuclear medium with transverse extent $d \gg b'_{\max}$ and the longitudinal extent $L \ll d$. This can be thought of as a prototype of an intense laser pulse. Any photon can be converted into an electric dipole at the expense of an additional α which, however, impacts only the total rate, but not the dynamics of the multiple scatterings. This motivated us to consider the photoproduction off the nuclear medium in more detail. Also, as has been mentioned, this case is similar in many aspects to the well-studied dynamics of the strong interactions.

At first, in Sec. V we study the Born approximation with the leading logarithmic accuracy. Born approximation corresponds to scattering on only one proton in a nucleus. For realistic nuclei, the photoproduction cross section reduces to the Bethe-Heitler formula (31) [14] with a characteristic Z^2 dependence on the proton number. In this case the elastic scattering amplitude $i\Gamma^{llp}$ is approximately real, see (25), corresponding to the dominance of elastic processes over the inelastic ones. This is not difficult to understand: a significant part of the cross section originates at distances much larger than the nuclear radius; the nuclear density there is small. On the contrary, in the case of the nuclear medium the amplitude $i\Gamma^{llp}$ is approximately imaginary; see (32) and (33c). This implies the dominance of inelastic processes. Formally, this happens because of an approximate cylindrical symmetry of the nuclear distribution around a small dipole on one hand, and antisymmetry of the real part of the scattering amplitude $\text{Re}[i\Gamma^{llp}]$ with respect to inversion $\mathbf{b} \rightarrow -\mathbf{b}$, see (12), on the other hand. For the nuclear medium, the photoproduction cross section is proportional to Z in the Born approximation.

Multiple rescatterings induce a correction to the Born approximation that generally depend on energy \sqrt{s} and nuclear charge Z . These effects are studied in Sec. VI. Expectedly, the multiphoton effects lead to rather different expressions for the resummed cross sections (40) and (43) for a single nucleus and for the nuclear medium. For a single nucleus the multiphoton processes resummed within the Glauber model do not depend on energy. In the Appendix, we demonstrate, using the method of [10], that the Glauber approach reproduces the result of Bethe and Maximon, i.e. unitarity corrections beyond the leading logarithmic approximation [3]. In the opposite case of the nuclear medium, the cross section grows as a function of energy and Z until it saturates the black disk geometric limit. A problem of propagation of positronium through

matter was previously discussed by Lyuboshits and Podgoretsky in Ref. [15] who reached similar conclusions.

A related problem has been recently addressed by Muller [16] who considered scattering of an intense laser pulse on a heavy nucleus. He argues—basing on work done with his collaborators [17]—that the resummation in the density of the laser beam can be phenomenologically significant if one studies the laser-nucleus interaction at the LHC.

II. PHOTOPRODUCTION IN THE DIPOLE MODEL

One of the Feynman diagrams for the photoproduction process is shown in Fig. 1. In the high energy limit $s \gg -t$, where s and t are the usual Mandelstam variables, the cross section for the diagram in Fig. 1 can be written as [18]

$$\begin{aligned} \sigma_{\text{tot}}^{\gamma Z}(s) &= \frac{1}{2} \int \frac{d^2 r}{2\pi} \int_0^1 d\zeta \Phi(\mathbf{r}, \zeta) \sigma_{\text{tot}}^{llZ}(\mathbf{r}, s) \\ &= \int d^2 r \int_0^1 d\zeta \Phi(\mathbf{r}, \zeta) \int d^2 B \text{Im}\langle i\Gamma^{llZ}(\mathbf{r}, \mathbf{B}, s) \rangle. \end{aligned} \quad (1)$$

This formula is the Fourier transform of the momentum space expression for the scattering amplitude. Dipole size \mathbf{r} is a separation of the l and \bar{l} in the transverse configuration space (i.e. in the plane perpendicular to the collision axes); this is the variable that is Fourier conjugated to the relative transverse momentum \mathbf{k} of the pair. We use the bold face to distinguish the transverse two-vectors. Impact parameter \mathbf{B} is a transverse vector from the nuclear center to the center of mass of the $l\bar{l}$ pair; see Fig. 2. In (1) $\Phi(\mathbf{r}, \zeta)$ is the square of the photon “wave function” averaged over the photon polarizations and summed over lepton helicities. It is given by [12,18]

$$\Phi(\mathbf{r}, \zeta) = \frac{2\alpha m^2}{\pi} \{K_1^2(rm)[\zeta^2 + (1 - \zeta)^2] + K_0^2(rm)\}, \quad (2)$$

where m is a lepton mass. ζ is the fraction of the photon light-cone momentum taken away by the lepton. $\langle i\Gamma^{llZ}(\mathbf{r}, \mathbf{B}, s) \rangle$ is the dipole-nucleus elastic scattering amplitude averaged over the proton positions (indicated by the

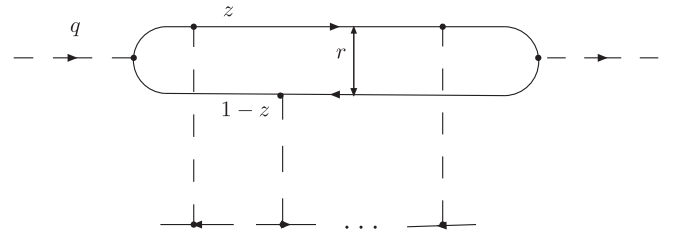


FIG. 1. Photoproduction of a lepton pair. Dashed lines denote photons, solid lines denote fermions. The ellipsis indicates summation over all proton numbers. Note that the t -channel photons can hook up to either fermion or antifermion lines.

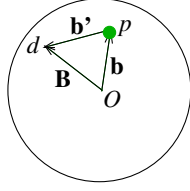


FIG. 2 (color online). Geometry of the dipole-nucleus collision in the transverse plane. p , d , and O denote proton, center of mass of the dipole, and the nucleus center positions correspondingly. $\mathbf{b}' = \mathbf{B} - \mathbf{b}$, $b \leq R$.

$\langle \cdots \rangle$ symbol) and the dipole-nucleus cross section is, by virtue of the optical theorem,

$$\sigma_{\text{tot}}^{\bar{l}Z}(\mathbf{r}, s) = 2 \int d^2B \text{Im} \langle i\Gamma^{\bar{l}Z}(\mathbf{r}, \mathbf{B}, s) \rangle. \quad (3)$$

Note, that in the high energy limit, the dipole cross section does not depend on ζ . Integrating over ζ we get

$$\sigma_{\text{tot}}^{\gamma Z}(s) = \frac{\alpha m^2}{\pi} \int \frac{d^2r}{2\pi} \left\{ \frac{2}{3} K_1^2(rm) + K_0^2(rm) \right\} \sigma^{\bar{l}Z}(\mathbf{r}, s). \quad (4)$$

Equations (1) and (2) admit the following interpretation. At high energies, the processes of the γZ scattering proceeds in two separated in time stages. The first stage is fluctuation of the photon into the lepton pair $\gamma \rightarrow l + \bar{l}$. Let the photon, lepton, and antilepton 4-momenta be $q = (q_+, 0, 0)$, $k = (\zeta q_+, k_-, \mathbf{k})$, and $k - q = ((1 - \zeta)q_+, (k - q)_-, -\mathbf{k})$, with $\zeta = k_+/q_+$. Here we use the light-cone momentum notation of the 4-vector $p^\mu = (p_+, p_-, \mathbf{p})$, where $p_\pm = p^0 \pm p^3$. The four-scalar product is $p \cdot k = \frac{1}{2}(p_+ k_- + p_- k_+) - \mathbf{p} \cdot \mathbf{k}$. The light-cone time span of this fluctuation is

$$\begin{aligned} \Delta x_+ &= \frac{1}{k_- + (q - k)_- - q_-} = \frac{1}{\frac{m^2 + \mathbf{k}^2}{\zeta q_+} + \frac{m^2 + \mathbf{k}^2}{(1 - \zeta)q_+}} \\ &= \frac{\zeta(1 - \zeta)q_+}{m^2 + \mathbf{k}^2}. \end{aligned} \quad (5)$$

The largest contribution to the cross section arises, as we will see later, from the longest possible Δx_+ , referred to as the coherence length l_c [19]. This corresponds to $\zeta = 1/2$ and $\mathbf{k} = 0$, so that $l_c = q_+/4m^2$. Obviously, $1/l_c = 4m^2/q_+$ is the minimum light-cone energy l_- transfer.

The second stage of the γZ process is interaction of the lepton pair with the nucleus. In the nucleus rest frame its time span is of the order of the target size $2R$, where R is the nucleus radius. We observe that $l_c \gg 2R$ for sufficiently high q_+ for a lepton of any mass m supporting the physical interpretation of the two stage process. It now becomes clear why it is convenient to write the cross section for the process Fig. 1 in a way in which the photon wave function describing the splitting process $\gamma \rightarrow l + \bar{l}$ is separated from the amplitude $\Gamma^{\bar{l}Z}$ describing interaction of

the $\bar{l}l$ electric dipole with the nucleus. Indeed such an approach proved very useful for the study of the strong interactions.

III. GLAUBER MODEL FOR MULTIPLE SCATTERING

Let the nucleus quantum state be described by the wave function ψ_A that depends on positions $\{\mathbf{b}_a, z_a\}_{a=1}^Z$ of all Z protons, where \mathbf{b}_a and z_a are the transverse and the longitudinal positions of proton a correspondingly. If the dipole-nucleus scattering amplitude $i\Gamma^{\bar{l}Z}$ is known, then averaging over the proton positions is performed as

$$\begin{aligned} \langle \Gamma^{\bar{l}Z}(\mathbf{r}, \mathbf{B}, s) \rangle &= \int \prod_{a=1}^Z d^2\mathbf{b}_a dz_a |\psi_A(\mathbf{b}_1, z_1, \mathbf{b}_2, z_2, \dots)|^2 \\ &\times \Gamma^{\bar{l}Z}(\mathbf{r}, \mathbf{B} - \mathbf{b}_1, z_1, \mathbf{B} - \mathbf{b}_2, z_2, \dots, s). \end{aligned} \quad (6)$$

The scattering amplitude is simply related to the scattering matrix element S as $\Gamma(s, \mathbf{B}) = 1 - S(s, \mathbf{B})$. The latter can in turn be represented in terms of the phase shift χ , so that in our case

$$\begin{aligned} \Gamma^{\bar{l}Z}(\mathbf{r}, \mathbf{B} - \mathbf{b}_1, z_1, \mathbf{B} - \mathbf{b}_2, z_2, \dots, s) \\ = 1 - \exp\{-i\chi^{\bar{l}Z}(\mathbf{r}, \mathbf{B} - \mathbf{b}_1, z_1, \mathbf{B} - \mathbf{b}_2, z_2, \dots, s)\}. \end{aligned} \quad (7)$$

At high energies, interaction of the dipole with different protons in the nucleus is independent inasmuch as the protons do not overlap in the longitudinal direction. This assumption is tantamount to taking into account only two-body interactions, while neglecting the many-body ones [2]. In this approximation the phase shift $\chi^{\bar{l}Z}$ in the dipole-nucleus interaction is just a sum of the phase shifts $\chi^{\bar{l}p}$ in the dipole-proton interactions, i.e.,

$$\begin{aligned} \Gamma^{\bar{l}Z}(\mathbf{r}, \mathbf{B} - \mathbf{b}_1, z_1, \mathbf{B} - \mathbf{b}_2, z_2, \dots, s) \\ = 1 - \exp\left\{-i \sum_a \chi^{\bar{l}p}(\mathbf{r}, s, \mathbf{B} - \mathbf{b}_a)\right\}. \end{aligned} \quad (8)$$

The problem of calculating the scattering amplitude of the dipole on a system of Z protons thus reduces to the problem of calculating of the scattering amplitude of the dipole on a single proton $\Gamma^{\bar{l}p}$. In this approximation, $\langle e^{-i\chi} \rangle = e^{-i\langle \chi \rangle}$, i.e. correlations between nucleons in the impact parameter space are neglected.

We can rewrite (6) and (7) as

$$\langle \Gamma^{\bar{l}Z}(\mathbf{r}, \mathbf{B}, s) \rangle = 1 - \left\langle \prod_a [1 - \Gamma^{\bar{l}p}(\mathbf{r}, s, \mathbf{B} - \mathbf{b}_a)] \right\rangle. \quad (9)$$

For a heavy nucleus with large Z we obtain

$$\langle \Gamma^{\bar{l}Z}(\mathbf{r}, \mathbf{B}, s) \rangle \approx 1 - \exp\{-Z \langle \Gamma^{\bar{l}p}(\mathbf{r}, s, \mathbf{B} - \mathbf{b}_a) \rangle\}. \quad (10)$$

This approximation is justified as long as $\alpha \ll 1$ and

$\alpha Z \sim 1$. Indeed, expression in the exponent of (10) can also be obtained by expanding

$$\begin{aligned} -i\chi^{\bar{l}Z} &= Z \ln(1 - \Gamma^{\bar{l}p}) \\ &\approx -Z[\Gamma^{\bar{l}p} - \frac{1}{2}(\Gamma^{\bar{l}p})^2 + \dots] \sim \alpha Z + \mathcal{O}(\alpha^2 Z). \end{aligned}$$

IV. DIPOLE-NUCLEUS SCATTERING AMPLITUDE

Now we turn to the calculation of the dipole-proton amplitude; see Fig. 3. At high energies the t -channel Coulomb photons are almost real: $l^2 \approx -\mathbf{l}^2$. This is the well-known Weizsäcker-Williams approximation that we use throughout the paper. It allows us to determine the leading logarithmic contribution to the scattering amplitude. In this approximation, the real part of the elastic dipole-proton scattering amplitude reads

$$\begin{aligned} \text{Re}[i\Gamma^{\bar{l}p}(\mathbf{r}, \mathbf{b}', s)] &= \frac{\alpha}{\pi} \int \frac{d^2 l}{l^2} [e^{i[\mathbf{b}' + (1/2)\mathbf{r}] \cdot \mathbf{l}} - e^{i[\mathbf{b}' - (1/2)\mathbf{r}] \cdot \mathbf{l}}] \\ &= 2\alpha \int_0^\infty \frac{dl}{l} \left[J_0\left(\left|\mathbf{b}' + \frac{1}{2}\mathbf{r}\right|l\right) - J_0\left(\left|\mathbf{b}' - \frac{1}{2}\mathbf{r}\right|l\right) \right]. \end{aligned} \quad (11)$$

Taking the integral we arrive at

$$\text{Re}[i\Gamma^{\bar{l}p}(\mathbf{r}, \mathbf{b}', s)] = 2\alpha \ln \frac{|\mathbf{b}' - \frac{1}{2}\mathbf{r}|}{|\mathbf{b}' + \frac{1}{2}\mathbf{r}|}. \quad (12)$$

The nuclear density $\rho(\mathbf{b}, z)$ is normalized such that

$$\int d^2 b dz \rho(\mathbf{b}, z) = Z. \quad (13)$$

Therefore, the average of the real part of the amplitude over the proton position reads

$$\langle \text{Re}[i\Gamma^{\bar{l}p}(\mathbf{r}, \mathbf{B}, s)] \rangle = \frac{1}{Z} \int d^2 b \int_{-z_0}^{z_0} dz \rho 2\alpha \ln \frac{|\mathbf{b}' - \frac{1}{2}\mathbf{r}|}{|\mathbf{b}' + \frac{1}{2}\mathbf{r}|}, \quad (14)$$

where $z_0 = 2\sqrt{R^2 - b^2}$. Since $\rho \approx \text{const}$, we get

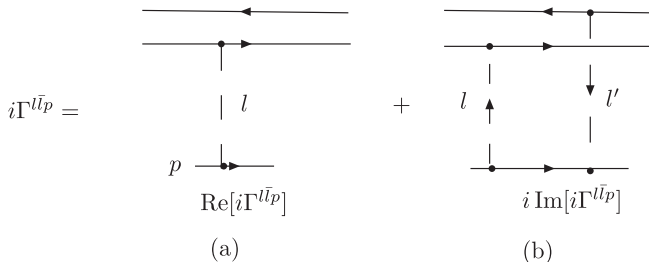


FIG. 3. The first two terms in the expansion of $\Gamma^{\bar{l}p}$ in α .

$$\langle \text{Re}[i\Gamma^{\bar{l}p}(\mathbf{r}, \mathbf{B}, s)] \rangle = \frac{2\alpha}{Z} \int d^2 b \rho 2\sqrt{R^2 - b^2} \ln \frac{|\mathbf{b}' - \frac{1}{2}\mathbf{r}|}{|\mathbf{b}' + \frac{1}{2}\mathbf{r}|}, \quad (15)$$

with $\mathbf{b}' = \mathbf{B} - \mathbf{b}$; see Fig. 2.

The imaginary part of the dipole-proton elastic scattering amplitude is depicted in Fig. 3(b). The corresponding analytical expression reads

$$\begin{aligned} \text{Im}[i\Gamma^{\bar{l}p}(\mathbf{r}, \mathbf{b}', s)] &= \frac{\alpha^2}{2\pi^2} \int \frac{d^2 l}{l^2} \int \frac{d^2 l'}{l'^2} \\ &\times (e^{i[\mathbf{b}' + (1/2)\mathbf{r}] \cdot \mathbf{l}} - e^{i[\mathbf{b}' - (1/2)\mathbf{r}] \cdot \mathbf{l}}) \\ &\times (e^{-i[\mathbf{b}' + (1/2)\mathbf{r}] \cdot \mathbf{l}'} - e^{-i[\mathbf{b}' - (1/2)\mathbf{r}] \cdot \mathbf{l}'}). \end{aligned} \quad (16)$$

Averaging over the proton positions we derive

$$\begin{aligned} \langle \text{Im}[i\Gamma^{\bar{l}p}(\mathbf{r}, \mathbf{B}, s)] \rangle &= \frac{1}{Z} \int d^2 b \rho 2\sqrt{R^2 - b^2} \frac{\alpha^2}{2\pi^2} \int \frac{d^2 l}{l^2} \\ &\times \int \frac{d^2 l'}{l'^2} (e^{i[\mathbf{b}' + (1/2)\mathbf{r}] \cdot \mathbf{l}} - e^{i[\mathbf{b}' - (1/2)\mathbf{r}] \cdot \mathbf{l}}) \\ &\times (e^{-i[\mathbf{b}' + (1/2)\mathbf{r}] \cdot \mathbf{l}'} - e^{-i[\mathbf{b}' - (1/2)\mathbf{r}] \cdot \mathbf{l}'}). \end{aligned} \quad (17)$$

The Weizsäcker-Williams approximation breaks down for very small momenta of the t -channel photons. To determine the cutoff transverse momentum l_{\min} at which this approximation still holds, we use the Ward identity applied to the lower vertex of Fig. 3(a). It gives $p \cdot l = p_+ l_- + p_- l_+ = 0$, where $p = (M^2/p_-, p_-, 0)$ with M being the proton mass. It follows that $l_+ = -p_+ l_- / p_-$. Thus, $-l^2 = -l_+ l_- + \mathbf{l}^2 \approx \mathbf{l}^2$ if $\mathbf{l}^2 \gg (p_+/p_-)l_-^2$. Since the minimum value of l_- is $4m^2/q_+$ [see (5) and the following text] we find

$$l_{\min} = \sqrt{\frac{p_+}{p_-} \frac{(2m)^4}{q_+^2}} = \sqrt{\frac{M^2(2m)^4}{p_-^2 q_+^2}} = \frac{M(2m)^2}{s}, \quad (18)$$

where $s = (p + q)^2 \approx p_- q_+$. l_{\min} corresponds to the maximum possible impact parameter $b'_{\max} \sim 1/l_{\min}$. As we will see in the next section, the energy dependence of the dipole-nucleus amplitude arises from the logarithmic dependence on these cutoff scales (this is why a precise value of the proportionality coefficient between b'_{\max} and $1/l_{\min}$ is not very important).

V. BETHE-HEITLER LIMIT

Before we consider a general case it is instructive to discuss the scattering process in the Born approximation. We would like to (i) confirm that we reproduce the result of Bethe and Heitler [14] and (ii) determine the range of impact parameters B that gives the largest contribution to the cross section.

The Born approximation corresponds to taking into account only the leading terms in the coupling α . Expanding the exponent in (10) we have

$$\langle \Gamma^{\bar{l}Z}(\mathbf{r}, \mathbf{B}, s) \rangle = Z \langle \Gamma^{\bar{l}p}(\mathbf{r}, \mathbf{B}, s) \rangle - \frac{1}{2} Z^2 \langle \Gamma^{\bar{l}p}(\mathbf{r}, \mathbf{B}, s) \rangle^2 + \dots \quad (19)$$

The real part of $i\Gamma^{\bar{l}p}$ stems from a diagram with a single photon exchange, while its imaginary part arises from the double photon exchange (in the high energy limit these diagrams are purely real and imaginary correspondingly); see Fig. 3. Therefore, at the lowest order in α , the imaginary part of $i\Gamma^{\bar{l}p}$ corresponds to the two photon exchange Fig. 3(b) that gives—according to (19)—contribution of order $\alpha^2 Z \sim \alpha$ to the cross section (3). On the other hand, the real part of $(i\Gamma^{\bar{l}p})^2$ corresponds (at the leading order in α) to the product of the two diagrams Fig. 3(a) with single photon exchanges; it is of the order $\alpha^2 Z^2 \sim 1$.¹ Thus, in the Born approximation

$$\sigma_{\text{tot},B}^{\bar{l}Z}(\mathbf{r}, s) = 2 \int d^2 B \left\{ \frac{1}{2} Z^2 \langle \text{Re}[i\Gamma^{\bar{l}p}(\mathbf{r}, \mathbf{B}, s)] \rangle^2 + Z \langle \text{Im}[i\Gamma^{\bar{l}p}(\mathbf{r}, \mathbf{B}, s)] \rangle \right\}. \quad (20)$$

The reason for keeping a subleading term in (20) (the term in the integrand proportional to Z) is that in the nuclear medium the real part of the dipole-proton amplitude vanishes, see Eq. (32), and then the main contribution stems from the imaginary part of that amplitude.

A. Single nucleus

Values of the dipole-proton impact parameter b' can range up to its maximal value b'_{max} , which is determined as the inverse minimal momentum transfer, i.e. $b'_{\text{max}} \sim 1/l_{\text{min}}$, where l_{min} is given by (18). For electron production at the collision energy of $\sqrt{s} = 10$ GeV we estimate $b'_{\text{max}} \sim 20$ nm, whereas the radius of the Gold nucleus is 7 fm. Thus for realistic nuclei $b'_{\text{max}} \gg R$. (In the case of τ photoproduction this approximation holds for energies larger than about $\sqrt{s} = 30$ GeV.)

We will demonstrate shortly that the large logarithmic contribution to the dipole cross section (20) comes from the region $b' \gg b$ and $b' \gg r/2$. Because by definition $b \leq R$, this is equivalent to $B \approx b' \gg \max\{R, \lambda/2\}$, where $\lambda = 1/m$ is the Compton wavelength of a lepton. Here we used the fact that the characteristic value of the dipole size is $r \sim 1/m$ since it corresponds to a sharp maximum of the integrand of (4). Note, that for e^-e^+ dipole $R \ll \lambda \ll b'_{\text{max}}$, while for μ and τ dipoles $\lambda \ll R \ll b'_{\text{max}}$.

¹This is in sharp contrast with QCD where the single gluon exchange amplitude is identically zero because the non-Abelian symmetry generators are traceless. Thus, there is no elastic scattering at this order. As a consequence the leading contribution in QCD is of the order of atomic weight A .

Using $b' \approx B \gg b$, see Fig. 2, we can write (15) as

$$\begin{aligned} \langle \text{Re}[i\Gamma^{e^-e^+p}(\mathbf{r}, \mathbf{B}, s)] \rangle &\approx 2\alpha \ln \frac{|\mathbf{B} - \frac{1}{2}\mathbf{r}|}{|\mathbf{B} + \frac{1}{2}\mathbf{r}|} \frac{1}{Z} \\ &\times \int d^2 b \rho 2\sqrt{R^2 - b^2} \\ &= 2\alpha \ln \frac{|\mathbf{B} - \frac{1}{2}\mathbf{r}|}{|\mathbf{B} + \frac{1}{2}\mathbf{r}|}. \end{aligned} \quad (21)$$

To obtain the cross section we need to integrate over the impact parameter \mathbf{B} . Expand the logarithm in (21) at $B \gg r/2$ as

$$\ln \frac{|\mathbf{B} - \frac{1}{2}\mathbf{r}|}{|\mathbf{B} + \frac{1}{2}\mathbf{r}|} \approx \frac{\mathbf{B} \cdot \mathbf{r}}{B^2} + \mathcal{O}(r^3/B^3), \quad (22)$$

we derive

$$\langle \text{Re}[i\Gamma^{e^-e^+p}(\mathbf{r}, \mathbf{B}, s)] \rangle \approx 2\alpha \frac{\mathbf{B} \cdot \mathbf{r}}{B^2}. \quad (23)$$

Turning to the imaginary part we can write Eq. (17) as follows:

$$\begin{aligned} \langle \text{Im}[i\Gamma^{e^-e^+p}(\mathbf{r}, \mathbf{B}, s)] \rangle &= \frac{\alpha^2}{2\pi^2} \int \frac{d^2 l}{l^2} \int \frac{d^2 l'}{l'^2} \\ &\times (e^{i[\mathbf{B} + (1/2)\mathbf{r}] \cdot \mathbf{l}} \\ &- e^{i[\mathbf{B} - (1/2)\mathbf{r}] \cdot \mathbf{l}})(e^{-i[\mathbf{B} + (1/2)\mathbf{r}] \cdot \mathbf{l}'} \\ &- e^{-i[\mathbf{B} - (1/2)\mathbf{r}] \cdot \mathbf{l}'})) \end{aligned} \quad (24a)$$

$$\begin{aligned} &= \frac{1}{2} 4\alpha^2 \ln^2 \frac{|\mathbf{B} - \frac{1}{2}\mathbf{r}|}{|\mathbf{B} + \frac{1}{2}\mathbf{r}|} \\ &\approx 2\alpha^2 \frac{(\mathbf{B} \cdot \mathbf{r})^2}{B^4}. \end{aligned} \quad (24b)$$

To the leading order in α we can neglect the small absorption effect given by (24b) and write for the scattering amplitude

$$\langle \Gamma^{\bar{l}p}(\mathbf{r}, \mathbf{B}, s) \rangle \approx i2\alpha \frac{\mathbf{B} \cdot \mathbf{r}}{B^2}. \quad (25)$$

Consequently, in the high energy and Born approximations the dipole-nucleus cross section becomes

$$\sigma_{\text{tot},B}^{\bar{l}Z}(\mathbf{r}, s) = Z^2 \int d^2 B 4\alpha^2 \frac{(\mathbf{B} \cdot \mathbf{r})^2}{B^4}. \quad (26)$$

Denote $\mathcal{R} = \max\{R, \lambda/2\}$. Integration over B in the interval $\max\{R, r/2\} < B < b_{\text{max}}$ gives

$$\sigma_{\text{tot},B}^{\bar{l}Z}(\mathbf{r}, s) = 4\pi\alpha^2 r^2 Z^2 \ln \frac{b_{\text{max}}}{\mathcal{R}}, \quad (27)$$

where the logarithmic accuracy allowed us replacement $r = 1/m$ in the argument of the logarithm. The total photoproduction cross section off a nucleus at the order α^3 in the perturbation theory reads

$$\begin{aligned}\sigma_{\text{tot},B}^{\gamma Z}(s) &= \sum_l \int \frac{d^2r}{2\pi} \int_0^1 d\xi \Phi(\mathbf{r}, \xi) \frac{1}{2} \sigma_{\text{tot},B}^{l\bar{l}Z}(\mathbf{r}, s) \\ &= \sum_l 4\alpha^3 Z^2 m_l^2 \int_0^\infty dr r \left[\frac{2}{3} K_1^2(rm_l) + K_0^2(rm_l) \right] \\ &\quad \times r^2 \ln \frac{b_{\text{max}}}{R},\end{aligned}\quad (28)$$

where the sum runs over all lepton species. Taking integrals over the dipole sizes finally produces

$$\sigma_{\text{tot},B}^{\gamma Z}(s) = \sum_l 4\alpha^3 Z^2 m_l^2 \left(\frac{2}{3} \frac{2}{3m_l^4} + \frac{1}{3m_l^4} \right) \ln \frac{b_{\text{max}}}{R}. \quad (29)$$

In particular, for electrons

$$\sigma_{\text{tot},B}^{\gamma Z \rightarrow e^+ e^- X}(s) = \frac{28}{9} \frac{\alpha^3 Z^2}{m_e^2} \ln \frac{s}{2Mm_e}. \quad (30)$$

In a frame where nuclear proton moves with velocity $v = (p_+ - p_-)/(p_+ + p_-)$, the minus component of its four-momentum is $p_- = M\sqrt{(1-v)/(1+v)}$. Therefore, $s = 2M\omega\sqrt{(1-v)/(1+v)}$, where the incoming photon frequency is $\omega = q_+/2$. Equation (30) now reads

$$\sigma_{\text{tot},B}^{\gamma Z \rightarrow e^+ e^- X}(s) = \frac{28}{9} \frac{\alpha^3 Z^2}{m_e^2} \ln \left(\frac{\omega}{m_e} \sqrt{\frac{1-v}{1+v}} \right). \quad (31)$$

This is the high energy limit of the formula derived by Bethe and Heitler [14]. In the Appendix we discuss the energy-independent correction to this equation.

B. Nuclear medium

Consider a nuclear matter having an arbitrary large number of protons Z . We assume that the interproton distance is much smaller than the Compton wavelength of electron, i.e. $\rho^{-1/3} \ll \lambda$. This allows treating the nuclear matter as the continuous medium of density ρ . Let the medium transverse size be $d \gg b'_{\text{max}}$ and longitudinal size $L \ll l_c$. In the medium rest frame $l_c \simeq b'_{\text{max}}$.² Therefore, the nuclear medium has a form of a film with $d \gg L$. What does the lepton photoproduction look like in this case? The key observation in this case is that the dipole-proton interaction range is much smaller compared to the transverse size of the medium. Therefore, $b' \ll B$ and then it follows from (15) that the real part of the elastic dipole-proton scattering amplitude vanishes. Indeed, changing the integration variable $\mathbf{b} \rightarrow \mathbf{b}'$ in (15) and taking the limit of small \mathbf{b}' we obtain

²The values of b'_{max} at $\sqrt{s} = 10$ GeV for e , μ , and τ are 20 nm, 0.4 pm, and 2 fm, correspondingly.

$$\begin{aligned}\langle \text{Re}[i\Gamma^{l\bar{l}p}(\mathbf{r}, \mathbf{B}, s)] \rangle &= \frac{2\alpha}{Z} \int d^2b' \rho(\mathbf{B} - \mathbf{b}') L \ln \frac{|\mathbf{b}' - \frac{1}{2}\mathbf{r}|}{|\mathbf{b}' + \frac{1}{2}\mathbf{r}|} \\ &\approx \frac{2\alpha}{Z} \rho(B) L \int d^2b' \ln \frac{|\mathbf{b}' - \frac{1}{2}\mathbf{r}|}{|\mathbf{b}' + \frac{1}{2}\mathbf{r}|} = 0.\end{aligned}\quad (32)$$

The last equation follows because the integrand is an odd function under the reflection $\mathbf{b}' \rightarrow -\mathbf{b}'$. The first nonvanishing contribution to the real part of the amplitude is proportional to the nuclear density gradient, which has the largest value at the boundary. These small effects can be taken into account if the nuclear density distribution is known.

The leading contribution arises from the imaginary part of the amplitude that reads

$$\begin{aligned}\langle \text{Im}[i\Gamma^{l\bar{l}p}(\mathbf{r}, \mathbf{B}, s)] \rangle &\approx \frac{1}{Z} \rho L \int d^2b' \frac{\alpha^2}{2\pi^2} \int \frac{d^2l}{l^2} \int \frac{d^2l'}{l'^2} \\ &\quad \times (e^{i[\mathbf{b}' + (1/2)\mathbf{r}] \cdot \mathbf{l}} - e^{i[\mathbf{b}' - (1/2)\mathbf{r}] \cdot \mathbf{l}}) \\ &\quad \times (e^{-i[\mathbf{b}' + (1/2)\mathbf{r}] \cdot \mathbf{l}'} - e^{-i[\mathbf{b}' - (1/2)\mathbf{r}] \cdot \mathbf{l}'})\end{aligned}\quad (33a)$$

$$= \frac{1}{Z} \rho L 4\alpha^2 \int \frac{d^2l}{l^4} (1 - e^{i\mathbf{r} \cdot \mathbf{l}}) \quad (33b)$$

$$\approx \frac{1}{Z} \rho L 4\alpha^2 (2\pi) \frac{1}{4} r^2 \ln \frac{2}{rl_{\text{min}}}, \quad (33c)$$

where in the last line we kept only the logarithmically enhanced term. We see that in the case of the nuclear medium, the entire elastic dipole-proton amplitude is approximately imaginary and is given by Eq. (33c). Moreover, unlike (24b) it increases logarithmically with energy. Substituting (32) and (33c) into (20) we obtain in the Born approximation

$$\sigma_{\text{tot},B}^{l\bar{l}Z}(\mathbf{r}, s) = 4\pi Z \alpha^2 r^2 \ln \frac{2}{rl_{\text{min}}}. \quad (34)$$

Note, that cross section (34) is Z times smaller than the one given by (27).

VI. UNITARITY EFFECTS

The unitarity relation applied to the elastic scattering amplitude at a given impact parameter reads [we suppress the argument $(\mathbf{r}, \mathbf{b}, s)$ of all functions]

$$2 \text{Im}(i\Gamma^{l\bar{l}Z}) = |\Gamma^{l\bar{l}Z}|^2 + G^{l\bar{l}Z}, \quad (35)$$

where $G^{l\bar{l}Z}$ is the inelastic scattering amplitude. Using (10) we can solve (35) as

$$G^{l\bar{l}Z} = 1 - e^{-Z \langle \text{Im}(i\Gamma^{l\bar{l}p}) \rangle}. \quad (36)$$

It follows that the total, inelastic and elastic cross sections are given by

$$\sigma_{\text{tot}}^{\bar{l}lZ} = 2 \int d^2B \{1 - \cos[Z\langle \text{Re}(i\Gamma^{\bar{l}l p}) \rangle]\} e^{-Z\langle \text{Im}(i\Gamma^{\bar{l}l p}) \rangle}, \quad (37a)$$

$$\sigma_{\text{in}}^{\bar{l}lZ} = \int d^2B \{1 - e^{-Z\langle \text{Im}(i\Gamma^{\bar{l}l p}) \rangle}\}, \quad (37b)$$

$$\sigma_{\text{el}}^{\bar{l}lZ} = \int d^2B \{1 - 2 \cos[Z\langle \text{Re}(i\Gamma^{\bar{l}l p}) \rangle]\} e^{-Z\langle \text{Im}(i\Gamma^{\bar{l}l p}) \rangle} + e^{-Z\langle \text{Im}(i\Gamma^{\bar{l}l p}) \rangle}. \quad (37c)$$

When $Z\langle \Gamma^{\bar{l}l p} \rangle \sim 1$, deviation from the Born approximation becomes large and the unitarity effects set in. We proceed to analyze the unitarity corrections in two extreme cases.

A. Single nucleus

In the case of photon-nucleus scattering, we derive using (25) and integrating over the directions of \mathbf{B} :

$$\begin{aligned} \sigma_{\text{tot}}^{\bar{l}lZ} &\approx \sigma_{\text{el}}^{e^-e^+Z} = 2 \int d^2B \left(1 - \cos\left[2Z\alpha \frac{\mathbf{B} \cdot \mathbf{r}}{B^2}\right]\right) \\ &= 4\pi \int_0^{B_{\text{max}}} dB B [1 - J_0(2Z\alpha r/B)]. \end{aligned} \quad (38)$$

Introducing the dimensionless variable $x = 2Z\alpha r/(2B_{\text{max}})$ we can express this integral in terms of the generalized hypergeometric function:

$$\begin{aligned} \sigma_{\text{tot}}^{\bar{l}lZ}(\mathbf{r}, s) &= \frac{(2Z\alpha r)^2 \pi}{8} \{x^2 {}_2F_3[(1, 1), (2, 3, 3), -x^2] \\ &\quad + 4(2 - 2\gamma - \ln x^2)\}, \end{aligned} \quad (39)$$

where γ is Euler's constant. The total photoproduction cross section of light leptons is obtained using (4). Actually, it is convenient to directly plug (38) into (4) and first integrate over r and then over B . The result can be expressed in terms of another dimensionless variable $y = \alpha Z/(B_{\text{max}} m)$ as

$$\begin{aligned} \sigma_{\text{tot}}^{\gamma Z}(s) &= \frac{\alpha^3 Z^2}{9m^2 y^3} (2y - 28y^3 \ln(2y) \\ &\quad + \{-3\sqrt{1+y^2} + 24y^2\sqrt{1+y^2} \\ &\quad + \cosh[3 \ln(y + \sqrt{1+y^2})]\} \ln(y + \sqrt{1+y^2})). \end{aligned} \quad (40)$$

To find the high energy asymptotic we expand (40) at small y and find

$$\sigma_{\text{tot}}^{\gamma Z}(s) = \frac{28\alpha^3 Z^2}{9m^2} \left[\ln \frac{1}{2y} + \frac{41}{42} + \frac{12}{35} y^2 + \dots \right]. \quad (41)$$

In the leading logarithmic approximation that we use in this paper, only the leading logarithm in the square brackets of (41) can be guaranteed. Indeed, it reproduces the Bethe-Heitler formula (30). However, Eq. (41) does allow us to conclude that corrections to the leading logarithm

must not increase with energy. Therefore, in the case of lepton pair photoproduction on a single nucleus the unitarity corrections do not grow with energy at the leading order in $\alpha \ll 1$ and $\alpha Z \sim 1$. Actually, the Glauber model that we use in this paper also allows us to reproduce the subleading (i.e. energy-independent) corrections to Eq. (30) that were first obtained by Bethe and Maximon [3]. This result can be easily derived using the formalism developed by Ivanov and Melnikov in [10]. The derivation is outlined in the Appendix. In the framework of our approach it is straightforward to calculate corrections of order $\alpha^2 Z$ to the Bethe-Maximon formula by taking into account the imaginary part of the amplitude (24b). This problem will be addressed elsewhere.

B. Nuclear medium

Conclusions of the previous section are dramatically reversed in the case of scattering off a nuclear medium. For the total dipole cross section we obtain after substituting (32) and (33c) into (37)

$$\sigma_{\text{tot}}^{\bar{l}lZ}(\mathbf{r}, s) = 2\pi d^2 \left(1 - \exp\left[-\rho L 2\pi \alpha^2 r^2 \ln \frac{2}{r l_{\text{min}}}\right]\right). \quad (42)$$

Similar results were obtained for an infinite medium in [15]. Substituting into (4) we derive

$$\begin{aligned} \sigma_{\text{tot}}^{\gamma Z}(s) &= 2\alpha m^2 d^2 \int \frac{d^2 r}{2\pi} \left\{ \frac{2}{3} K_1^2(rm) + K_0^2(rm) \right\} \\ &\quad \times [1 - e^{-\rho L 2\pi \alpha^2 r^2 \ln(2/r l_{\text{min}})}]. \end{aligned} \quad (43)$$

At large Z the multiphoton effects start to play an important role.

It is instructive to study the unitarity corrections by increasing the proton number Z while keeping the medium size given by L and d fixed. This corresponds to an increase in the proton number density. We are interested to know at what Z 's the multiphoton effects become observable. In

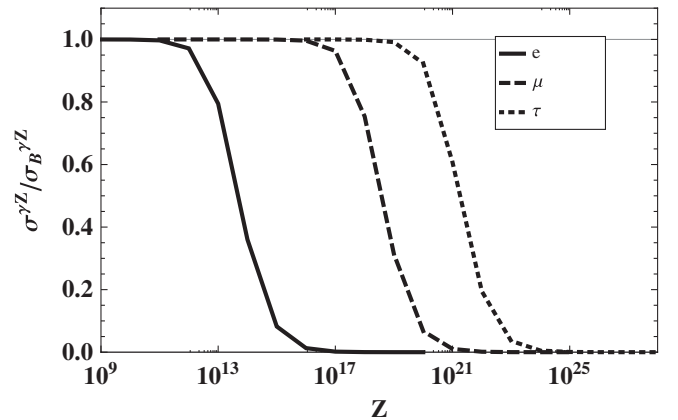


FIG. 4. Ratio of the lepton photoproduction cross section and its Born approximation as a function of the “nucleus” charge Z . $\sqrt{s} = 10$ GeV, $L = 1$ nm, and $d = 100$ nm.

Fig. 4 we show the ratio of the total photoproduction cross calculation to its Born approximation for $\sqrt{s} = 10$ GeV and $L = 1$ nm, $d = 100$ nm. We observe that for electrons the deviation from the linear Born regime starts at about $Z \sim 10^{12}$ charges. At $Z \sim 10^{13}$ the cross section is essentially black (i.e. Z independent).

VII. CONCLUSIONS

In this article we discussed an effect of multiple scattering on the high energy lepton photoproduction off a heavy nucleus and nuclear medium. The heavy nucleus serves as a source of strong Coulomb field. Resummation of multiple scattering of a lepton dipole in the nucleus amounts to resummation of diagrams with an arbitrary number of photons attached to the lepton loop. In the heavy nucleus case we selected only diagrams that are enhanced by the power of Z and resummed them using the Glauber model. We found that dominance of elastic scattering leads to energy-independent unitarity corrections. These corrections do not saturate the geometric limit of the cross sections. The unitarity corrections that grow with energy can appear as a result of multiple lepton pair production at very high energies [20]. The dipole model that we used in this paper is a convenient tool for taking into account such high energy evolution effects. We plan to address this problem elsewhere.

Lepton photoproduction off a nuclear medium is different in many aspects. Here the processes are predominantly inelastic. At very large Z the cross sections (37) saturate at the geometric limit due to multiphoton interactions with the medium. An additional contribution to the saturation stems from the multiple electron-positron pair production [20,21]. This leads to growth of the imaginary part of $i\Gamma^{lp}$ as a power of energy s^Δ with $\Delta \approx (11/32)\pi\alpha^2$ [21]. However, since Δ is very small this effect is mostly of academic interest. Parametrically, lepton photoproduction off the nuclear medium is very similar to the hadron photoproduction in strong interactions. The real part of $i\Gamma^{lp}$ vanishes identically in that case because it is proportional to the vanishing trace of the Gell-Mann matrices. In QCD, the quark-antiquark color dipole interacts with the heavy nucleus by means of multiple exchange of virtual gluons. Additionally, multiple gluon emission contributes to the power-law dependence of the cross section. The corresponding power is not small leading to important observable effects. Unitarity correction in QCD grows very large at high energy leading to saturation of the scattering amplitudes [22–24]. In lepton photoproduction QCD evolution will arise at high orders of perturbation theory.

In the case of the nuclear medium, weakness of the EM coupling can be compensated by a large number density of charges. In a more realistic model one should also take into account screening of the Coulomb fields by electrons leading to the emergence of the nuclear form factor. However, our emphasis in this article is the study of

electromagnetic interactions in a high density system. A high density of charges can be realized also in ultrafast pulse lasers that emit pulses containing as many as $N \sim 10^{20}$ photons and having dimensions $L < d$. In this case the inelastic in-medium effects start to play a significant role in a large range of collision energies. Although fluctuation of laser photons into electric dipoles is suppressed by the power of α , the created Coulomb field is enhanced by large effective charge Z . We are going to address this intriguing problem in a forthcoming publication.

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APPENDIX: DERIVATION OF THE BETHE-MAXIMON FORMULA

Equation (38) is written in the leading logarithmic approximation that does not allow one to reproduce the subleading energy-independent terms in the total cross section. However, the Glauber model allows one to calculate the total cross section with better accuracy. To this end, one has to abandon the approximation of (22) and use Eq. (21) for the real part of the amplitude in (37). We have

$$\sigma_{\text{tot}}^{\gamma Z}(s) = \frac{\alpha m^2}{\pi} \int \frac{d^2 r}{2\pi} \left[\frac{2}{3} K_1^2(mr) + K_0^2(mr) \right] 2 \int d^2 B \times \left\{ 1 - \cos \left[2\alpha Z \ln \frac{|\mathbf{B} - \mathbf{r}/2|}{|\mathbf{B} + \mathbf{r}/2|} \right] \right\}. \quad (\text{A1})$$

Introducing a new integration variable $\mathbf{R} = \mathbf{B} + \mathbf{r}/2$ we rewrite (A1) as

$$\sigma_{\text{tot}}^{\gamma Z}(s) = \frac{\alpha m^2}{\pi} \int \frac{d^2 r}{2\pi} \left[\frac{2}{3} K_1^2(mr) + K_0^2(mr) \right] 2 \int d^2 R \times \left\{ 1 - \frac{1}{2} \left(\frac{|\mathbf{R} - \mathbf{r}|}{R} \right)^{2i\alpha Z} - \frac{1}{2} \left(\frac{|\mathbf{R} + \mathbf{r}|}{R} \right)^{-2i\alpha Z} \right\}. \quad (\text{A2})$$

Consider

$$\Delta\sigma_{\text{tot}}^{\gamma Z} = \sigma_{\text{tot}}^{\gamma Z} - \sigma_{\text{tot},B}^{\gamma Z}, \quad (\text{A3})$$

i.e. the contribution due to multiple rescatterings. The term corresponding to the Born approximation is obtained by expanding the expression in the curly brackets in (A2) at small αZ . One then arrives at the formula derived by Ivanov and Melnikov [10]. It reads

$$\Delta\sigma_{\text{tot}}^{\gamma Z} = 2 \frac{\alpha m^2}{\pi} \int \frac{d^2 r}{2\pi} \left[\frac{2}{3} K_1^2(mr) + K_0^2(mr) \right] \int d^2 R \times \left\{ 1 - \frac{1}{2} \left(\frac{|\mathbf{R} - \mathbf{r}|}{R} \right)^{2i\alpha Z} - \frac{1}{2} \left(\frac{|\mathbf{R} - \mathbf{r}|}{R} \right)^{-2i\alpha Z} - 4(\alpha Z)^2 \ln^2 \frac{|\mathbf{R} - \mathbf{r}|}{R} \right\}. \quad (\text{A4})$$

Taking integrals over \mathbf{R} and \mathbf{r} yields

$$\Delta\sigma_{\text{tot}}^{\gamma Z} = -\frac{28}{9m^2} \alpha^3 Z^2 \frac{1}{2} [\psi(1 - i\alpha Z) + \psi(1 + i\alpha Z) - 2\psi(1)], \quad (\text{A5})$$

where $\psi(z)$ is the logarithmic derivative of the Γ function. As expected, the unitarity corrections are independent of energy in agreement with the results of Sec. VI A.

To calculate the energy-independent term of the Born approximation [the last term in the curly brackets of (A4)],

one needs to take into account the fact that the smallest longitudinal momentum transfer depends on ζ and \mathbf{k} ; see (5). In the logarithmic approximation, we maximized (5) with respect to both ζ and \mathbf{k} . Now it must be retained which means that (i) the regularization of the logarithmically divergent integrals must be done in momentum space and (ii) the ζ integral [see (1)] must be done after the regularization since the cutoff depends on it. We refer the interested reader to Ref. [10] for details. The result is

$$\sigma_{\text{tot},B}^{\gamma Z}(s) = \frac{28}{9} \frac{\alpha^3 Z^2}{m_e^2} \left(\ln \frac{s}{Mm_e} - \frac{109}{42} \right). \quad (\text{A6})$$

The second term in the brackets is the subleading correction we were looking for. The final result is the sum of (A5) and (A6) in agreement with [3].

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